Simulating and investigating compressible flows interaction with fractal structures

Omar Es-Sahli

Department of Aerospace Engineering, Mississippi State University

Adrian Sescu, Mohammed Afsar , Oliver R.H. Buxton

Department of Aerospace Engineering, Mississippi State University

Department of Mechanical & Aerospace Engineering, Strathclyde University, Glasgow, UK

Department of Aeronautics, Imperial College London, London, UK

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❑ Outline

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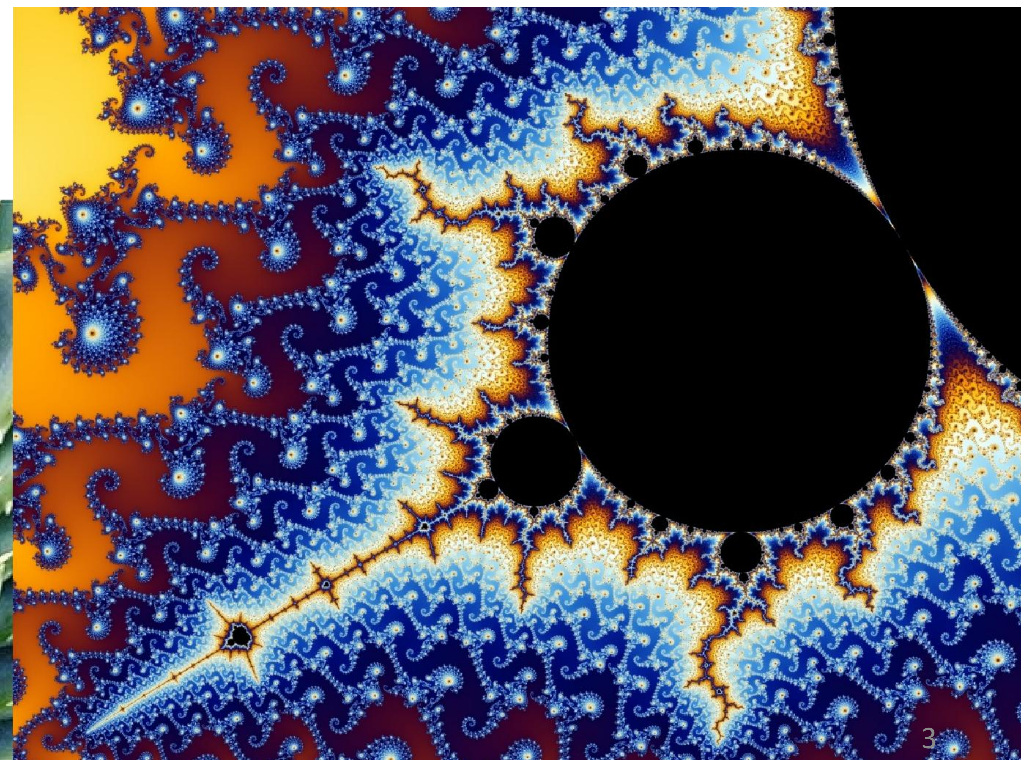
  - Motivation
  - Wake generators / fractal plates considered in this study
- Problem formulation and numerical algorithm
  - Scaling / non-dimensionalization
  - Numerical framework
  - Immersed boundary method
- Results & discussion
- Conclusion

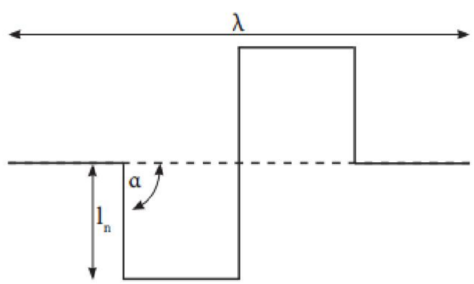
❑ Introduction

o What is a fractal?

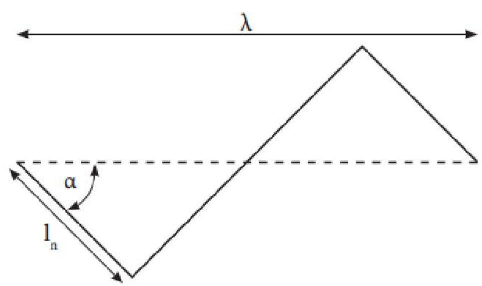
- A fractal is a detailed, recursive, and infinitely self-similar mathematical set that exhibits similar patterns at increasingly small scales.
- In other words and in the most basic sense, fractals are objects that display self-similarity over a wide range of scales.
- Introduced by Mandelbrot to extend the concept of theoretical fractional dimensions to geometric patterns found in nature.



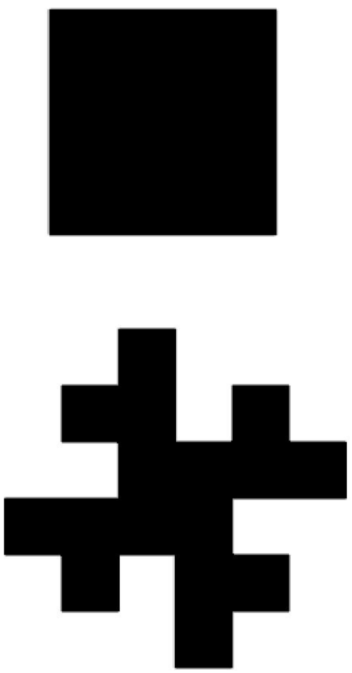
- Example: Consider a straight line segment of length λ. For the first iteration (n=1) with a square pattern(α =90°), λ is replaced by d = 8 segments of length $l_1 = \lambda / r$. For n iterations, the length of the segment is $l_n = l_{(n-1)}/r$.



(a) $D_f = 1.5$



(b) $D_f = 1.3$

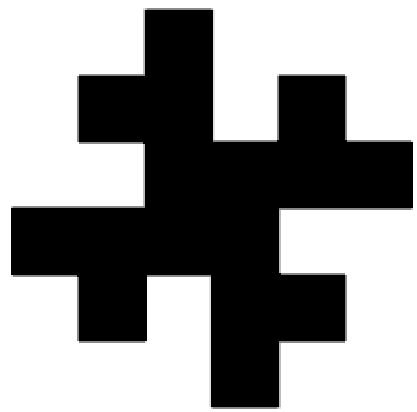


○ Motivation

- Previous studies
- Incompressible
- Result in higher turbulence intensities and a more enhanced turbulent mixing.
- Reduce the impact of the recirculation region around aircraft parts, e.g. spoilers, and hence the low-frequency noise.
- Significantly changes the near-field structure of the jet (by breaking up the large-scale coherent structures) responsible for the low-frequency noise.
- The mathematical properties of some fractals.
- Area conservation

- Wake generators / fractal plates considered in this study

(a) Square



(b) $D_f 1.5(1)$

(c) $D_f 1.5(2)$

❑ Governing equations & numerical framework

○ Scaling / non-dimensionalization

- All dimensional spatial coordinates are normalized by the reference length D associated with the fractal geometry.

$$(x, y, z) = \frac{(x^*, y^*, z^*)}{D}$$

- The velocity is scaled by the freestream velocity magnitude V_∞^*

$$(u, v, w) = \frac{(u^*, v^*, w^*)}{V_\infty^*}$$

- The pressure and temperature are non-dimensionalized, respectively, by the freestream dynamic pressure $\rho_\infty^* V_\infty^{*2}$ and temperature T_∞^*.

$$M_a = \frac{V_\infty^*}{a_\infty^*}\ , \quad Re_\lambda = \frac{\rho_\infty^* V_\infty^* D}{\mu_\infty^*}\ , \quad P_r = \frac{\mu_\infty^* C_p}{k_\infty^*}$$

- where a_∞^*, μ_∞^*,k_∞^* stand for, respectively, the freestream speed of sound, dynamic viscosity and thermal conductivity, C_p the specific heat at constant pressure.

- Full compressible Navier-Stokes equations in generalized curvilinear coordinates

○ Numerical framework

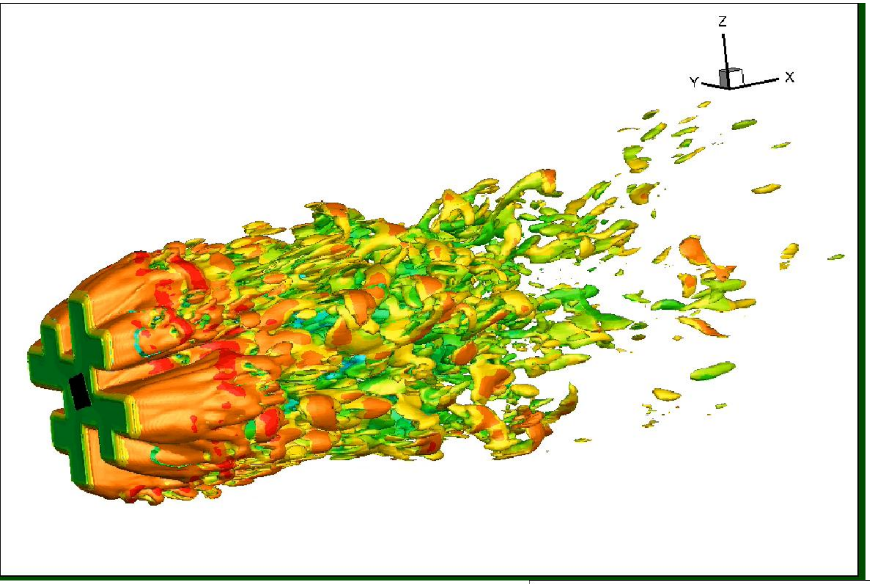
- Implicit large eddy simulations, where numerical filtering is applied to account for the missing sub-grid scale energy.
- The numerical algorithm uses high-order finite difference approximations for the spatial derivatives and explicit time marching.
- The time integration is performed using a third order TVD Runge-Kutta method.

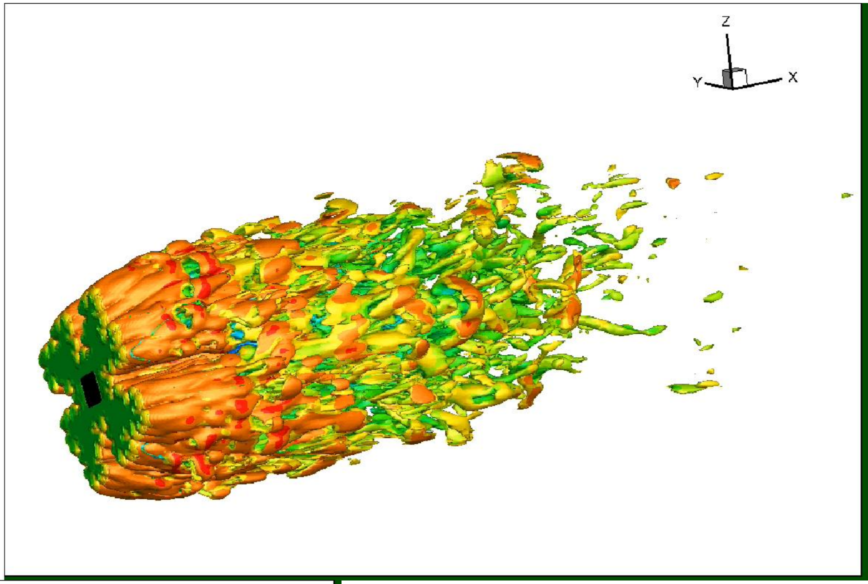
○ Immersed boundary method

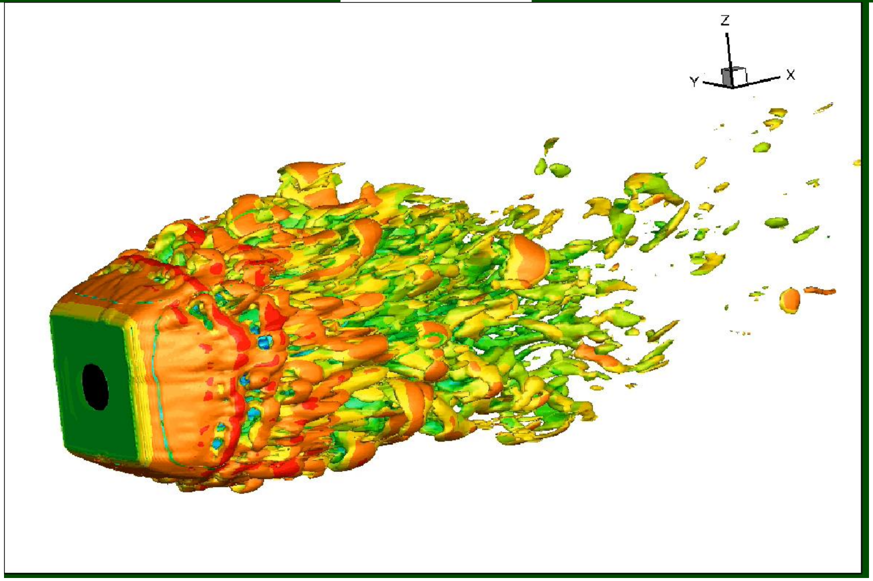
- the construction of the solid geometry inside the Cartesian grid is achieved by adding a forcing term **f** to the momentum equations that represents the impermeability of the fractal geometry to the governing equations.
- The fractal objects are obtained by multiple geometrical constrains. The forcing term consists of a penalty factor σ multiplied by the difference between the conserved variables $\rho, \ \rho u_i, \ and \ E$ and the imposed ones $\rho_{imp}, \ \rho u_{i,imp}, \ E_{imp}$.

❑Results

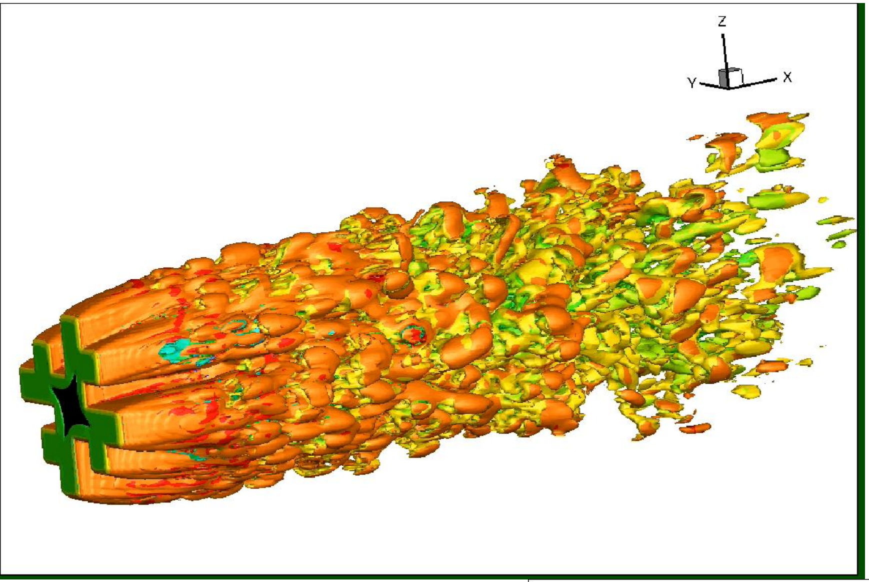
M = 0.2 Iso-surfaces of the vorticity magnitude colored by the velocity magnitude

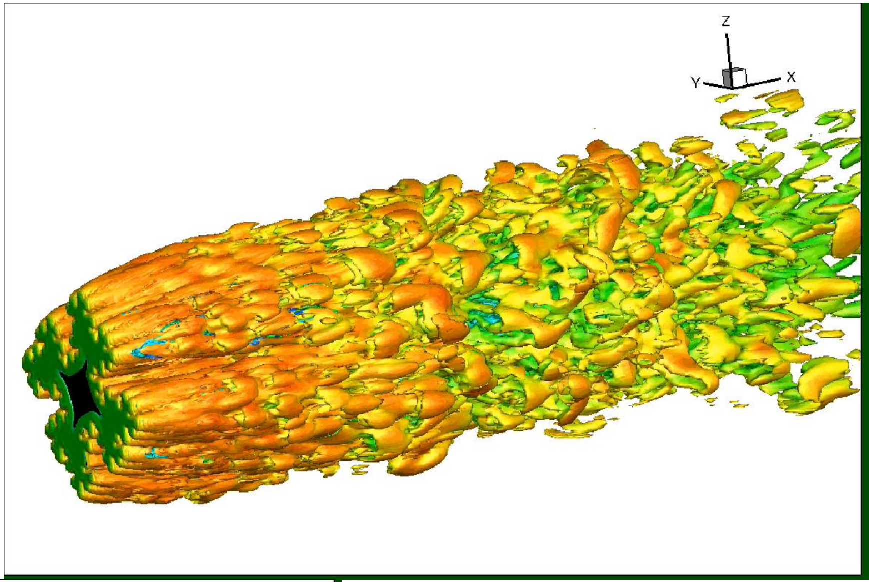


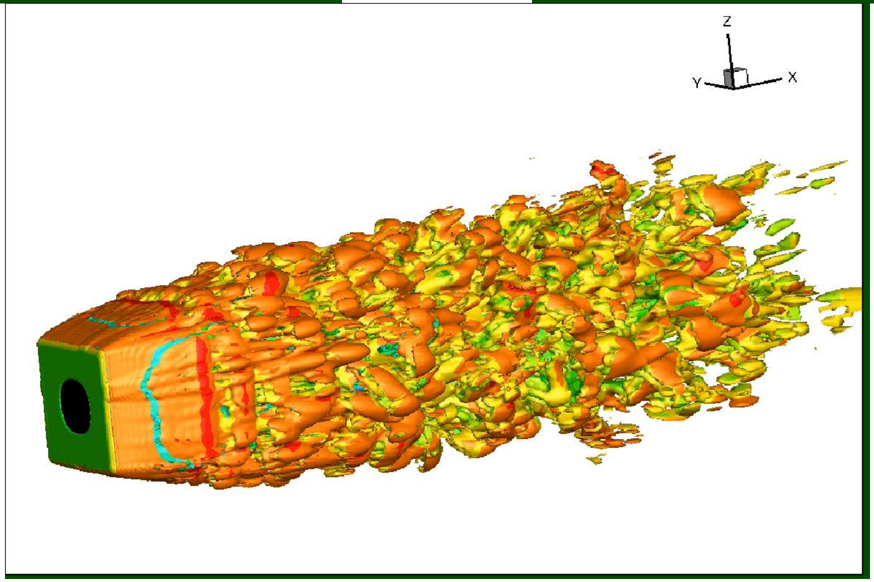




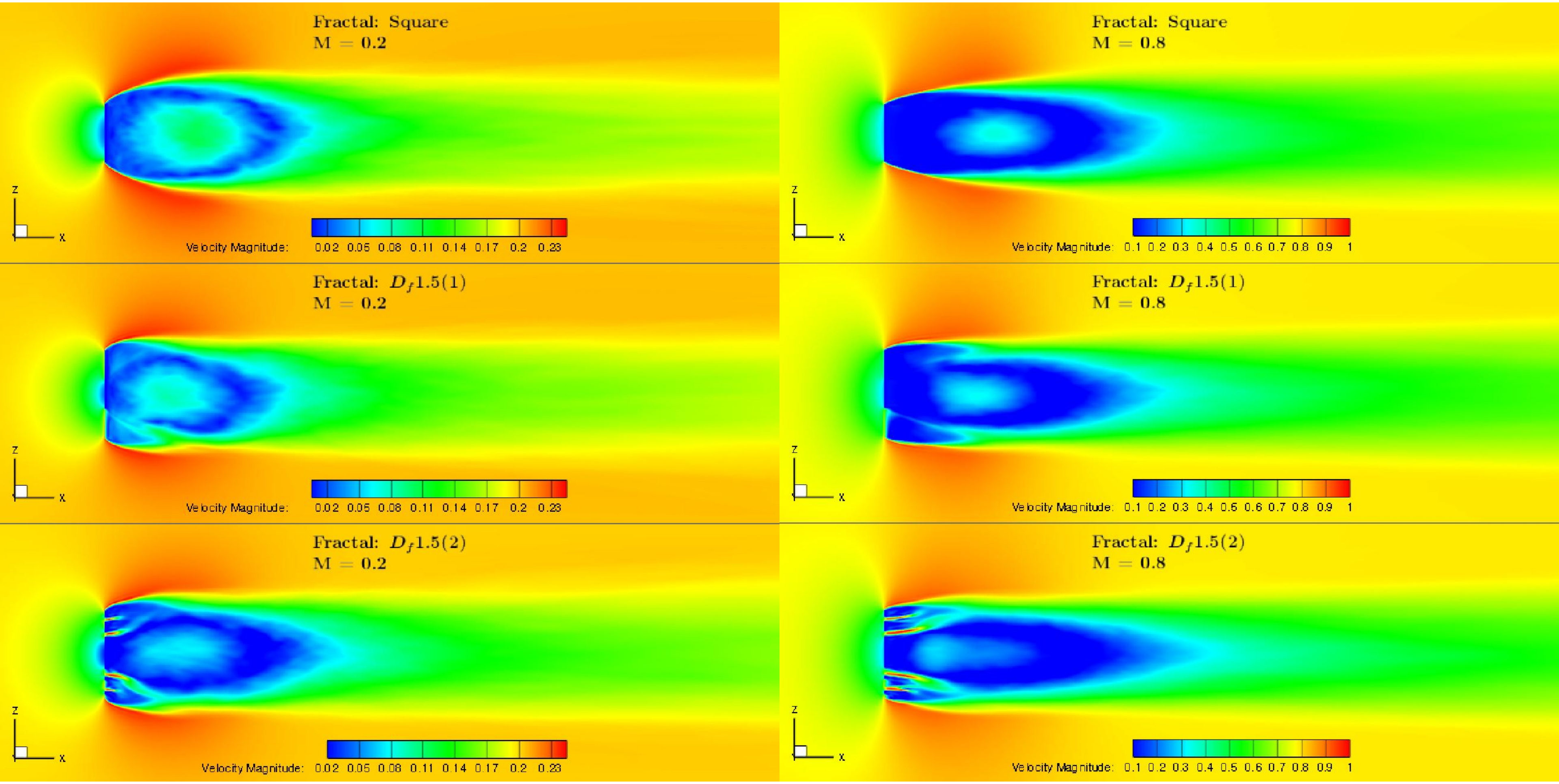
M = 0.8 Iso-surfaces of the vorticity magnitude colored by the velocity magnitude



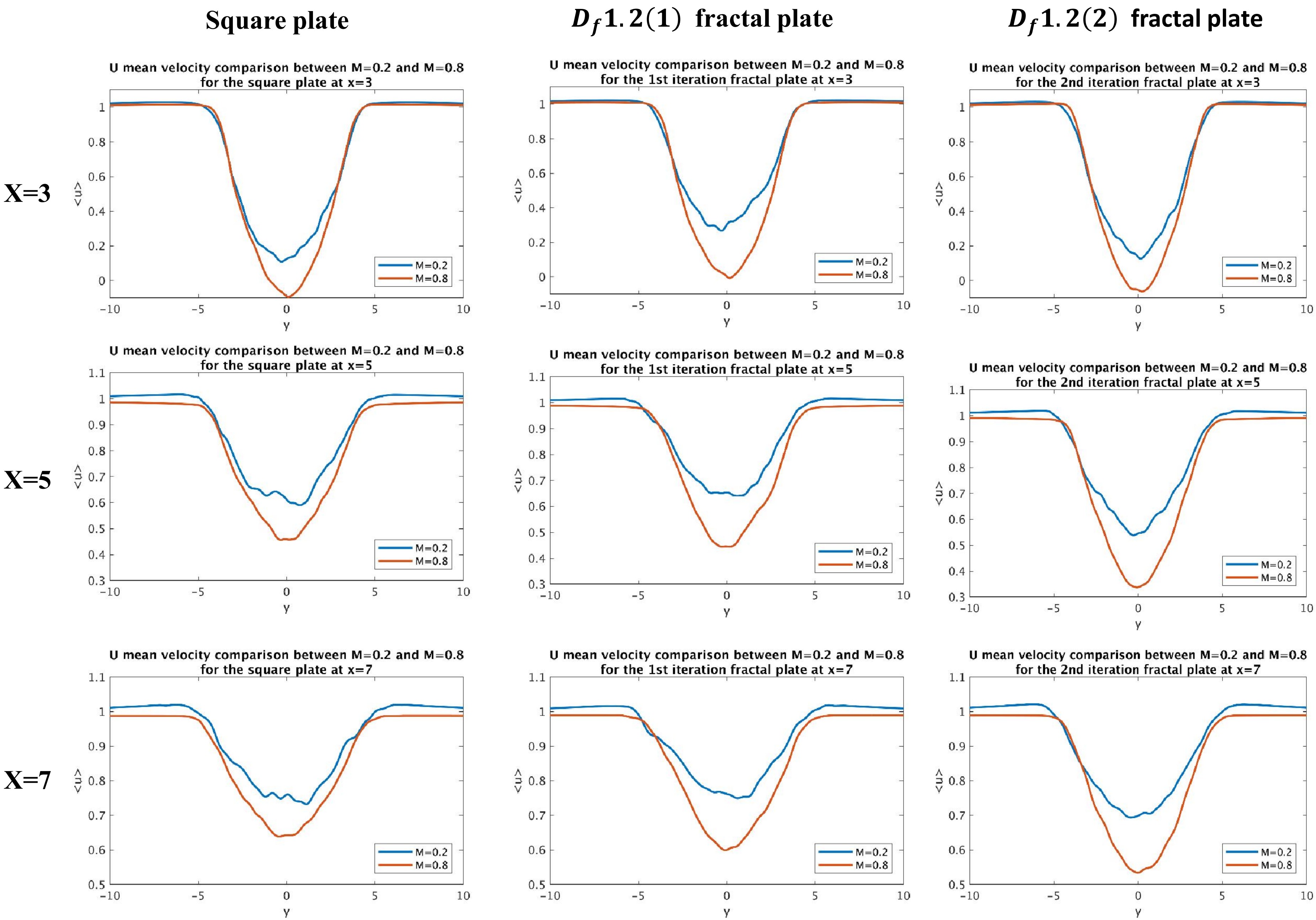




Contour plots of the XZ-plane



U mean velocity



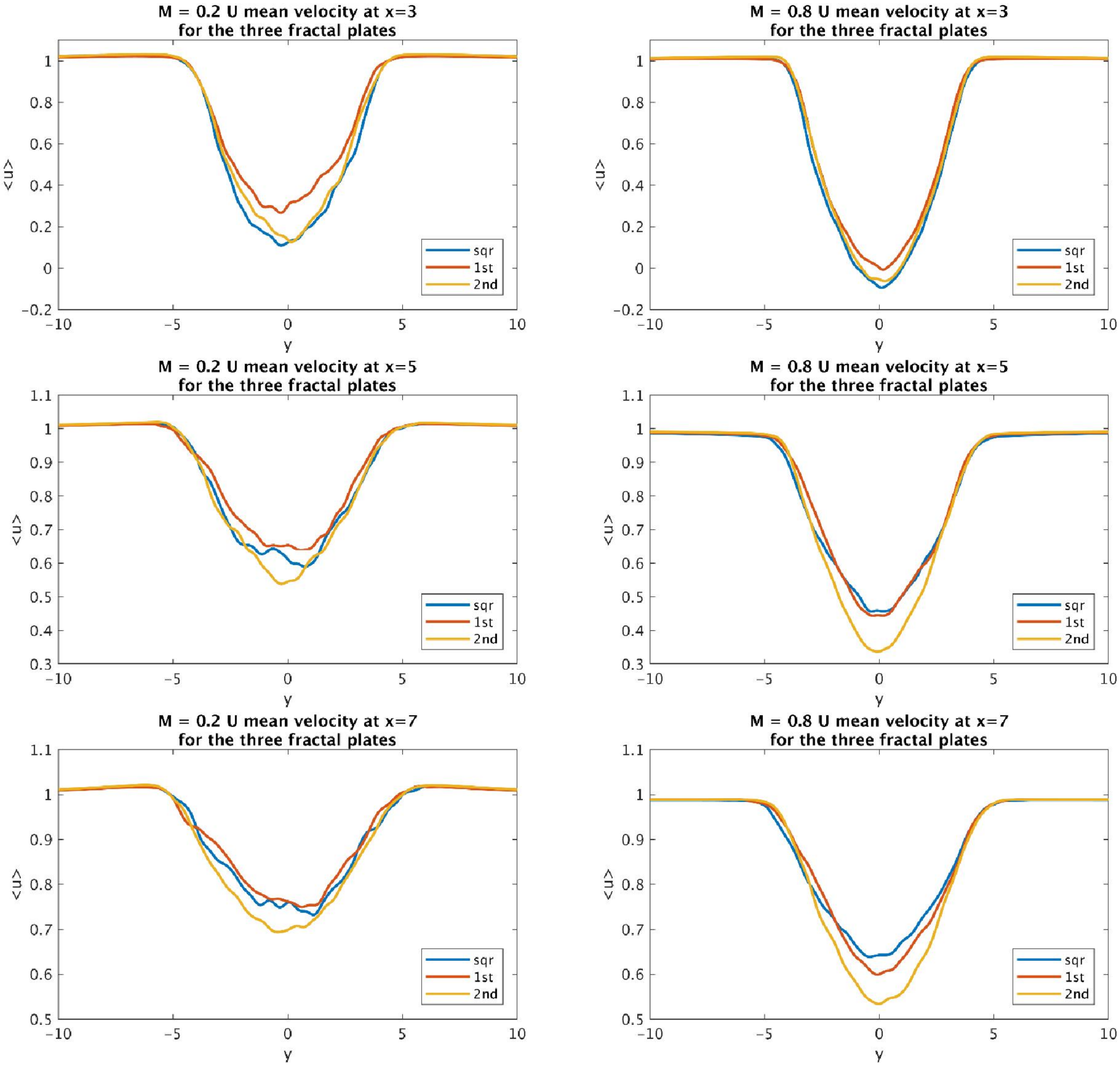
U mean velocity

M = 0.2 **M = 0.8**

X=3

X=5

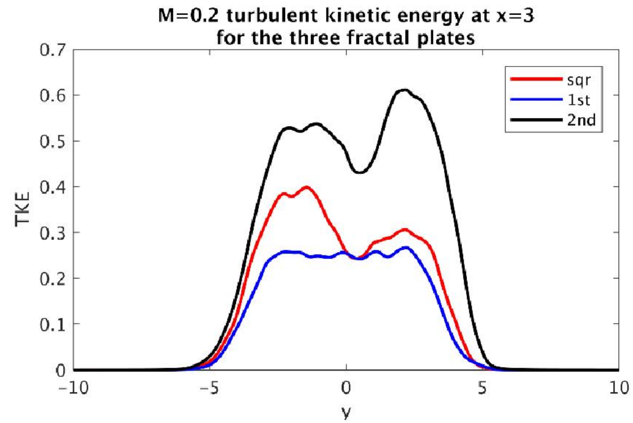
X=7

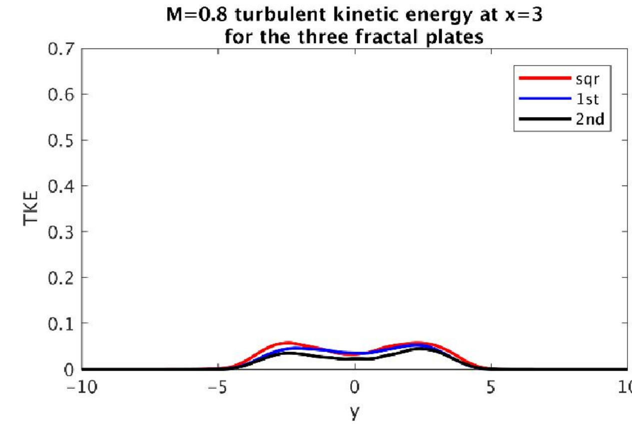


Turbulent kinetic energy

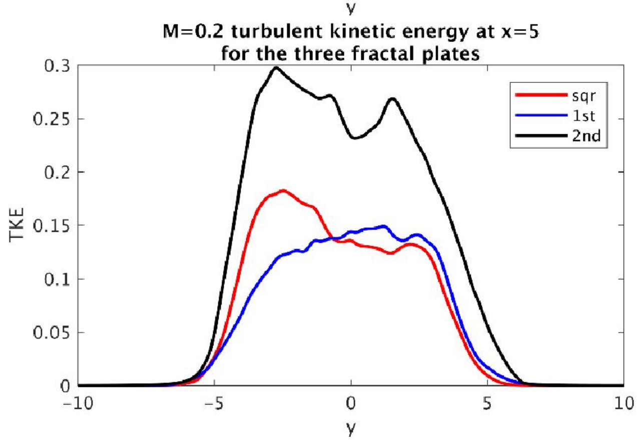
M = 0.2 **M = 0.8**

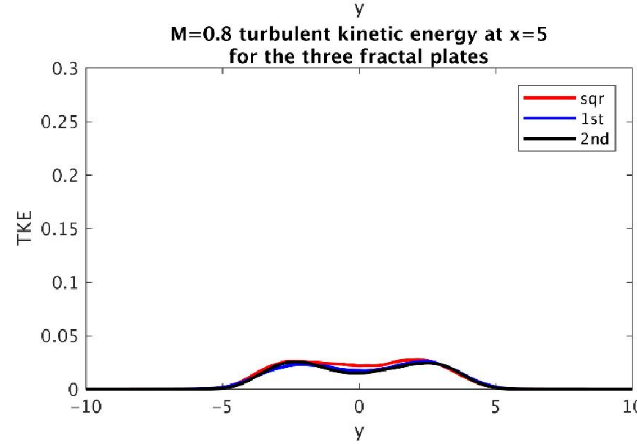
X=3



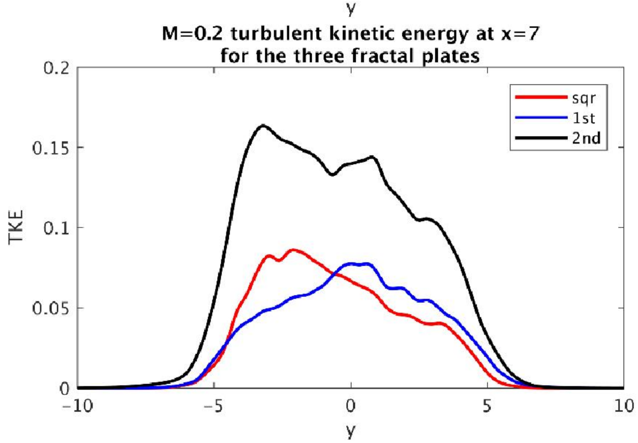


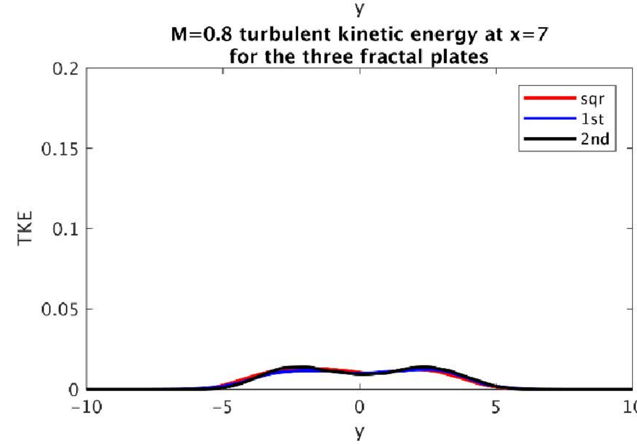
X=5





X=7

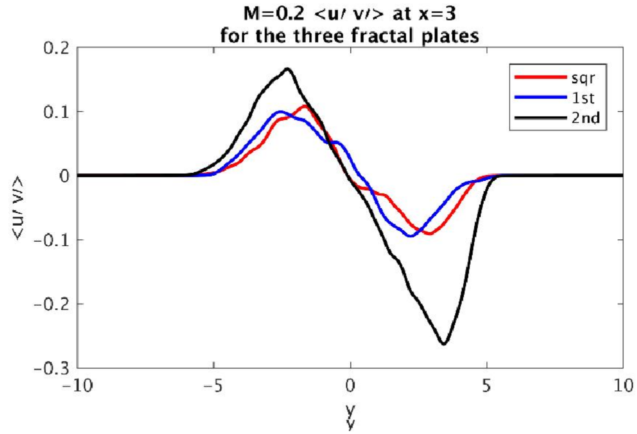


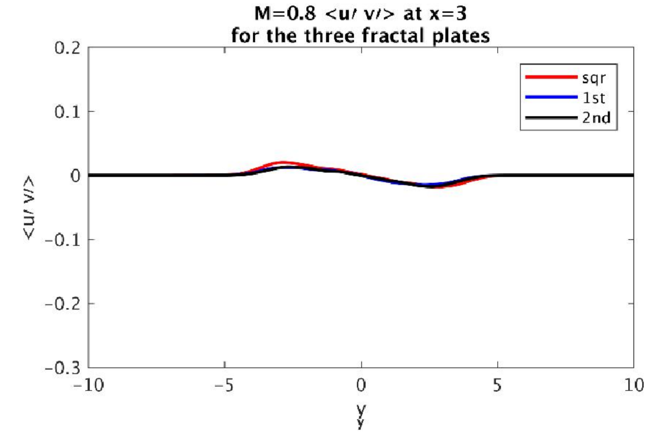


u’v’

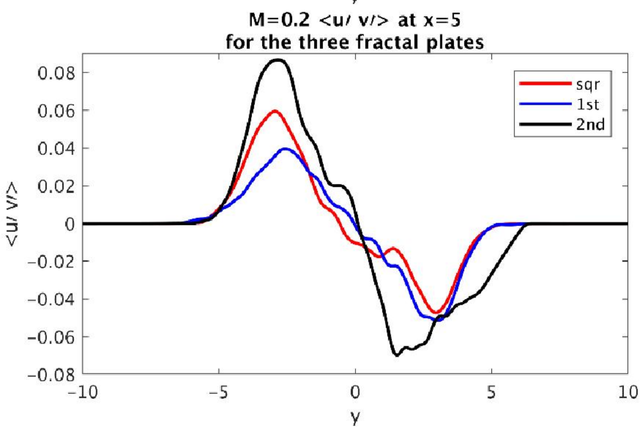
M = 0.2 | **M = 0.8**

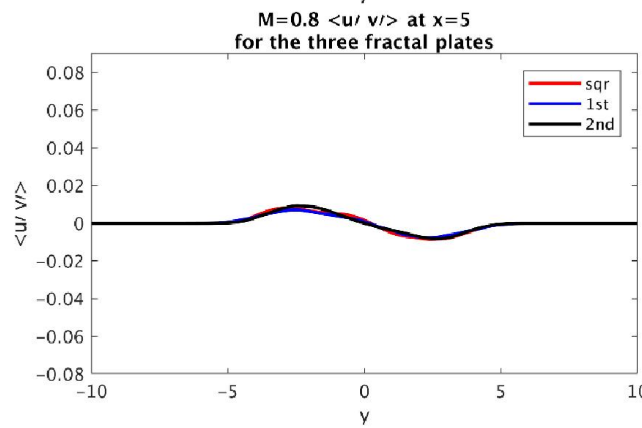
X=3



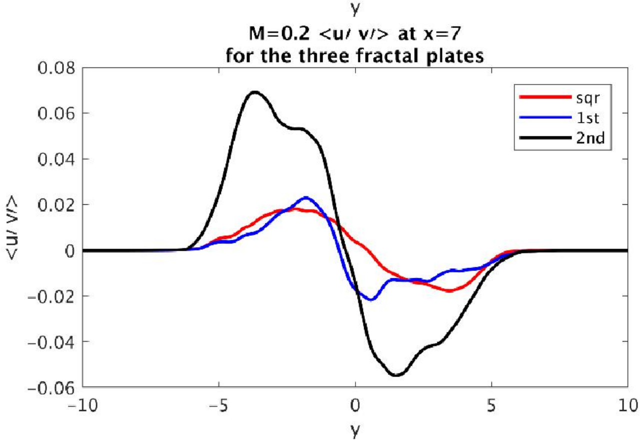


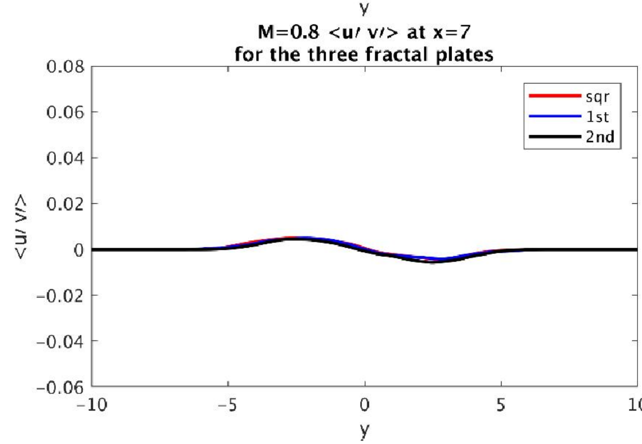
X=5





X=7

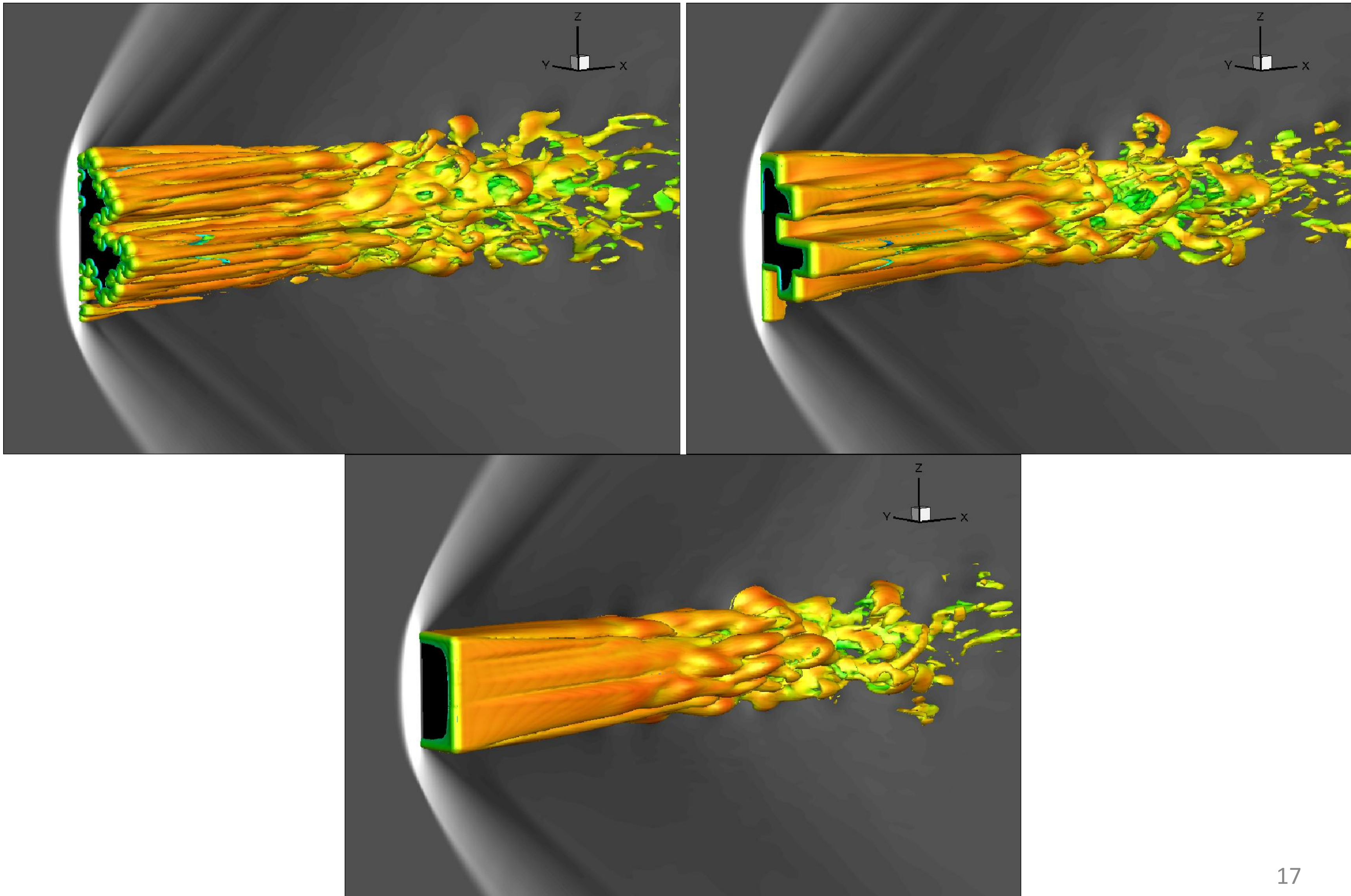




❑ Conclusion

- Ongoing work
  - Higher Mach numbers
- Future work
  - Jets

M = 1.5 Iso-surfaces of the vorticity magnitude colored by the velocity magnitude



Thank you.
Questions?